MBC-003-001105 Seat No. ________

First Year B. Sc. (Sem. I) (CBCS) Examination

November / December – 2016

Mathematics : Paper - BSMT - 101 (A)

(Theory) (Geometry & Calculus)

Faculty Code : 003
Subject Code : 001105

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions:-

1) All the questions are compulsory.

2) Numbers written to the right indicate full marks of the question.

1. Answer all the following 20 short answer questions. [20]

(1) Vector equation of the sphere having center at $(0,0,0)$ and radius a is ________.

(2) Write statement of the Cauchy's mean value theorem.

(3) When we say that f is bounded function ?

(4) The nth derivative of $y=e^{-x}$ is ________.

(5) Obtain the Cartesian co-ordinate for the polar co-ordinate $\left(1,\frac{\pi}{2}\right)$.

(6) If $\lim_{x\to 0}\frac{\sin 2x+a\sin x}{x^3}=b$ then find the value of a & b.

(7) Find n^{th} derivative of $\cos(ax+b)$.

(8) Evaluate $\int\cos^6 x dx$.

(9) The reduction formula for $\int_0^{\frac{\pi}{4}}\sin^4 2x dx=\frac{3\pi}{16}$ is True or False .

(10) Write Maclaurin's infinite series.

(11) Give the Expansion of sinhx in terms of x .

(12) (cosy+ycosx)dx+(sinx-xsiny)dy=0 is differential equation of the type____________.

(13) $(1-D)^{-1}=$___________.

(14) If $m_1=m_2$ are two real equal roots of the equation f(D)=0 then C.F.is ________.

(15) For the differential equation Mdx+Ndy=0 , if Mx+Ny≠0 and the equation is homogeneous then integrating factor is__________.

(16) $\frac{1}{D^2}\sin 2x =$_______.

(17) $f(x)=x^2+1, if\, 0\le x\le 1$

$= -\frac{x}{2}+2, if\, 1\le x\le 2$ then Rolle's theorem is applicable or not ? Justify your answer?

(18) $\frac{1}{D-a}X$ = ________.

(19) What is General form of Linear Differential Equation?

(20) Solve : $xdy+ydx=0$

2 (a) Attempt any **THREE**. [06]

(1) Convert the polar equation into Cartesian equation: $r\cos\theta = 2\sin^2\frac{\theta}{2}$.

(2) Give relation between Spherical co-ordinates and Cartesian co-ordinates.

(3) Show that: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - ....$

(4) Evaluate $\lim_{x\to a}\frac{\log(x-a)}{\log(e^x-e^a)}$ using L-Hospital's rule.

(5) If $y=\log(ax+b)$ then find y_n.

(6) State the Rolle's theorem.

(b) Attempt any **THREE**. [09]

(1) Find the center and radius of the sphere $x^2+y^2+z^2-2x-4y-6z-11=0$.

(2) Evaluate $\lim_{x\to 0}(\frac{a^x+b^x+c^x}{3})^{\frac{1}{x}}$ using L-Hospital's rule.

(3) Show that $\frac{2}{\pi} < \frac{\sin x}{x} < 1, 0 < x < \frac{\pi}{2}$.

(4) Verify Rolle's theorem for the function $f(x)=\frac{\sin x}{e^x}$ in the interval $(0,\pi)$.

(5) Find n^{th} derivative of $\cos x \cos 2x \cos 3x$.

(6) Expand e^x in ascending powers of $(x-2)$.

(c) Attempt any **TWO**. [10]

(1) State and prove Lagrange's mean value theorem.

(2) State L-Hospital's rule for $\frac{0}{0}$ form. Using this evaluate

$\lim_{x\to 0}\frac{xe^x-\log(1+x)}{x^2}$.

(3) If $y=\sin(m\sin^{-1}x)$ then show that

$(1-x^2)y_{n+2}-(2n+1)xy_{n+1}+(m^2-n^2)y_n=0$.

(4) State and prove Leibnitz's theorem.

(5) A Sphere of constant radius k passes through the origin and meets the axes in A,B,C. Prove that the centroid of ΔABC lies on the sphere $9(x^2+y^2+z^2)=4k^2$.

3 (a) Attempt any **THREE**. [06]

(1) Give order and degree of differential equation $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}=c$.

(2) Define linear differential equation.

(3) Solve: $y=px-\frac{1}{3}p^2$.

(4) Find the integrating factor of the differential equation $x\frac{dy}{dx}-y=2x^2$.

(5) Write reduction formula for $\int\sin^n x dx$.

(6) Find the complementary function of differential equation $\frac{d^2y}{dx^2}-7\frac{dy}{dx}+12y=e^{5x}$.

(b) Attempt any **THREE**. [09]

(1) Prove that $\frac{1}{f(D)}X$ is the particular integral of the equation $f(D)Y=X$.

(2) Solve: $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+13y=0$.

(3) Define Bernoulli's differential equation and solve $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$.

(4) Solve: $\frac{d^3y}{dx^3}-7\frac{dy}{dx}-6y=0$.

(5) Find $\frac{1}{D^3}(5x^2)$.

(6) Evaluate $\int_{0}^{\frac{\pi}{6}} \cos^6 3\theta \sin^3 6\theta d\theta$.

(c) Attempt any **TWO**. [10]

(1) Solve: $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 2y = 7e^x$.

(2) Derive the reduction formula of $\int \sin^m x \cos^n x dx$.

(3) Solve: $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x$.

(4) Prove that the necessary and sufficient condition for the differential equation $M(x,y)dx + N(x,y)dy = 0$ to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

(5) Solve: $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$.